MATHEMATICS

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Research-Validated Practices for Mathematics Instruction

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RESEARCH-VALIDATED PRACTICES FOR MATHEMATICS INSTRUCTION

In the United States, at Grades 4, 8, and 12, the majority of students do not meet minimum levels of proficiency in mathematics (Nation's Report Card, 2022). Core instruction and any intervention efforts need to rely on research-validated practices when students experience difficulty with mathematics. A *research-validated practice* is one that has been shown, through multiple high-quality research studies, to lead to improved mathematics outcomes for students.

When teachers implement research-validated practices, student mathematics outcomes often improve (Jitendra et al., 2018; Powell, Doabler, et al., 2020; Rojo et al., 2024; Stevens et al., 2018). This is true across grade levels, from the elementary grades through middle and high school, and across content areas of mathematics (Cook et al., 2020; Ennis & Losinski, 2019). Because of the cumulative nature of mathematics, in which earlier mathematics is essential for later mathematics, mathematics instruction using research-validated practices at any grade level is necessary, particularly if students experience difficulty with mathematics.

Across mathematics studies focused on instructional practices, several research-validated practices have emerged (Fuchs, Newman-Gonchar, et al., 2021). In this paper, we describe four such practices: **explicit instruction, mathematics vocabulary instruction, fluency practice,** and **problem solving**. Each of these research-validated practices could be incorporated into any mathematics curriculum.

Explicit Instruction

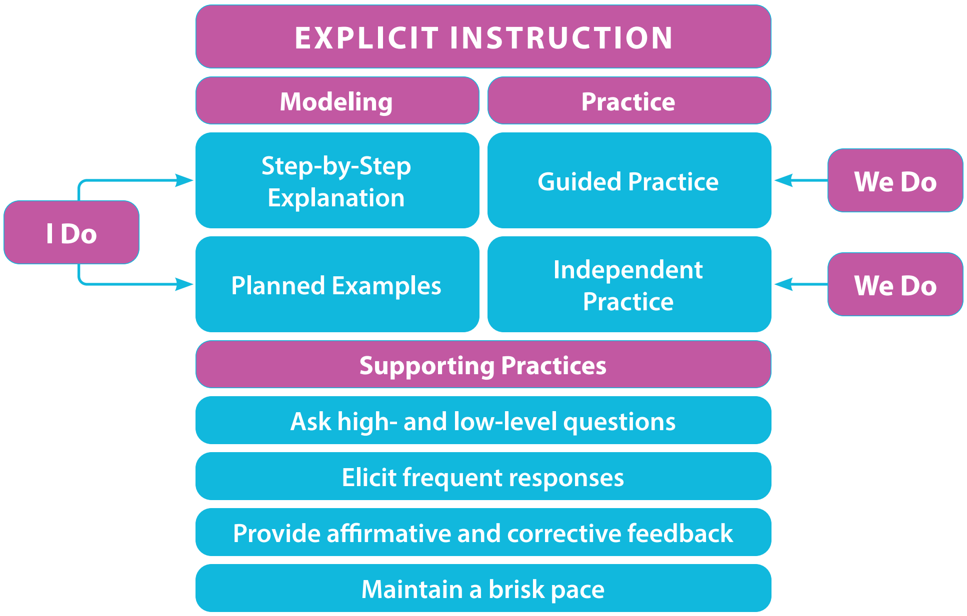
Explicit instruction is an evidence-based instructional design and strategy that provides a clear framework for delivering systematic and effective instruction (Hughes et al., 2017). We define *explicit instruction* as clear and concise instruction that occurs within teacher modeling and student practice opportunities. Explicit instruction is an essential component of mathematics instruction and enhances learning opportunities for students in both in-person and virtual learning environments (Bouck et al., 2021; Fuchs, Newman-Gonchar, et al., 2021; Powell, Mason, et al., 2021).

Research Supporting the Practice

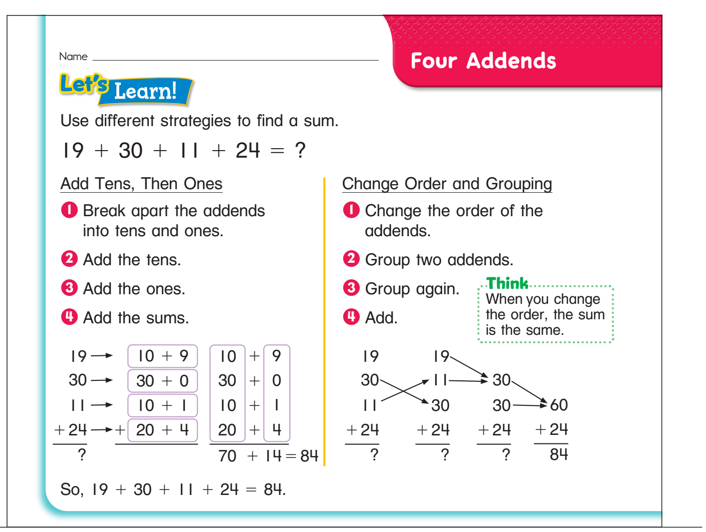
The What Works Clearinghouse and National Center on Intensive Intervention (Fuchs, Newman-Gonchar, et al., 2021; Schumacher et al., 2017) have identified explicit instruction as an essential instructional approach for students experiencing difficulty with mathematics. In elementary mathematics studies, researchers indicated explicit instruction is a significant component for improving students' counting and cardinality, place value, and addition and subtraction understanding, as well as understanding of fractions and word-problem solving (Bryant et al., 2008; Doabler et al., 2015; Fuchs, Schumacher et al., 2013; Powell, Berry, et al., 2020).

Similarly, in secondary mathematics instruction, explicit instruction is crucial for students to enhance their mathematical skills in problem solving and algebra. Researchers have identified explicit instruction as one of the effective practices for teaching algebra to students with learning difficulties, highlighting its role in improving educational outcomes (Gersten et al., 2009; Watt & Therrien, 2016). In a recent synthesis with middle school students, authors deemed explicit instruction as one of the most used instructional practices, supporting its effectiveness in improving algebraic skills among students with mathematics difficulty (Lariviere, Powell, et al., 2024). This approach offers clarity and reduces cognitive load, allowing students to concentrate on developing understanding of mathematical concepts and procedures.

Explicit Instruction in Action



The first element of explicit instruction is teacher-led modeling. Teachers explain a clear goal and model the activity for the students. When introducing any mathematics content, teachers will demonstrate step-by-step explanations and use clear language to solve the problem. For instance, teachers can model how to count the total number in a set (i.e., cardinality), add 26 and 18, identify common denominators between fractions, solve word problems, or calculate the area of a rectangle. Teachers should use planned examples in which easier



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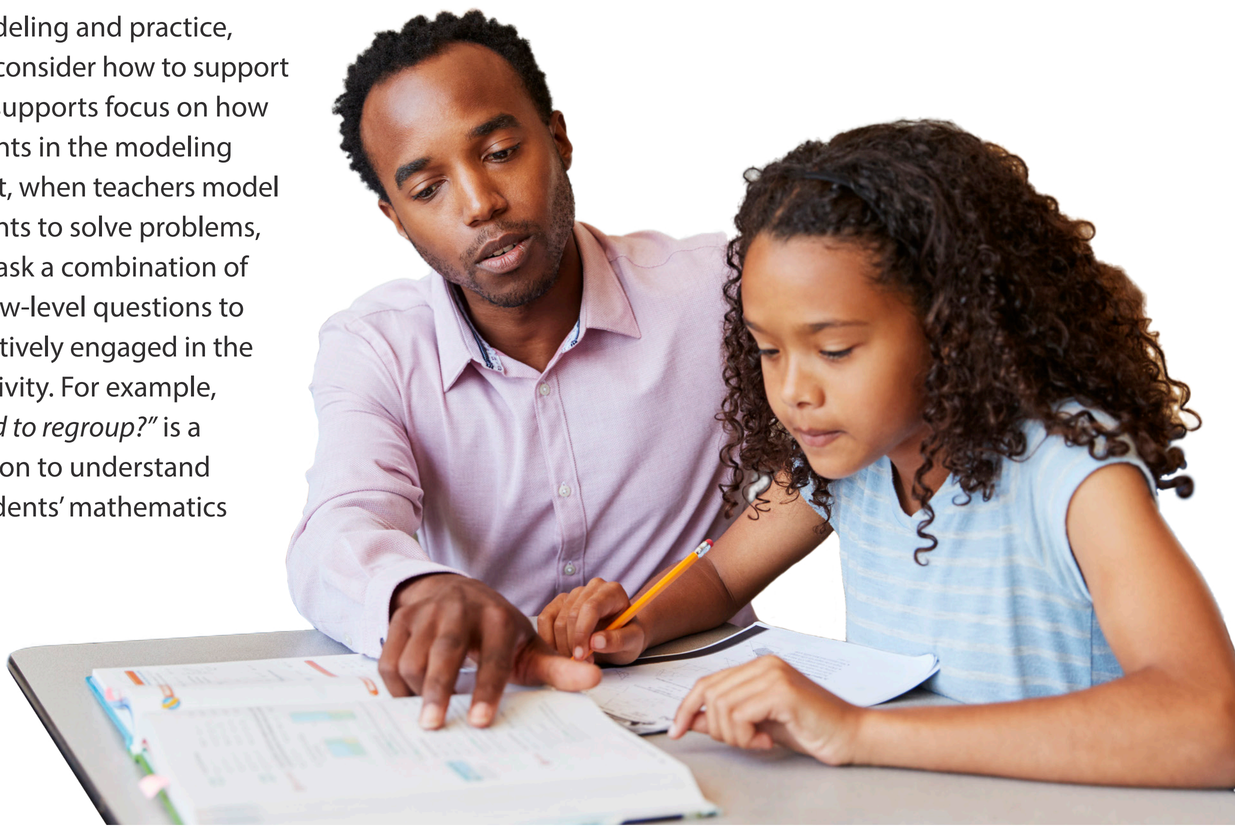
When introducing any mathematics content, teachers will demonstrate step-by-step explanations and use clear language to solve the problem.

problems lead to more complex problems. The number of modeled problems may vary based on students' needs (Doabler & Fien, 2013). For example, teachers might model only one or two problems for content that is easier for students.

Teacher-led modeling supports students to understand the steps of how to solve mathematics problems, but practice allows students to engage with the problems and learn how to solve the problems. During guided practice, teachers solve the problems together with the students. For example, when the teacher subtracts 18 from 26 and starts with the ones place using base-10 blocks, students also will use base-10 blocks and complete the same actions with the teacher. Guided practice supports students through scaffolding by providing prompts for the steps to solving a specific type of problem. Some students only may need to solve a few problems within guided practice whereas other students may require more guided practice problems.

When students show a strong knowledge of how to solve a specific type of mathematics problem, they should participate in independent practice. If possible, when students practice solving mathematics problems independently, teachers should provide feedback to students to ensure that students have developed strong foundational knowledge for a specific type of problem.

During both modeling and practice, teachers should consider how to support students. These supports focus on how to engage students in the modeling and practice. First, when teachers model and guide students to solve problems, teachers should ask a combination of high-level and low-level questions to keep students actively engaged in the mathematics activity. For example, *"Why do you need to regroup?"* is a high-level question to understand the depth of students' mathematics



content. High-level questions often start with *"why"* or *"how."* Low-level questions usually include *"what," "when,"* or *"which"* (e.g., *"Which number is a factor?"*). Second, teachers should ask such questions frequently to encourage students' engagement with the modeling and practice. In mathematics instruction, students should often respond—at a minimum—every 30–60 seconds during teacher modeling and guided practice.

Third, teachers should provide affirmative and corrective feedback based on students' responses. For example, *"I notice you followed all the steps correctly to solve this subtraction problem"* is affirmative feedback to help students understand that they are on the right track. Teachers should use the corrective feedback if students make errors and have misconceptions about mathematics content, such as *"Let's practice regrouping together one more time."* Finally, teachers can prepare and organize the lesson materials ahead of time to maintain a brisk pace to aid with appropriate pacing of the lesson.

Mathematics Vocabulary Instruction

In schools, mathematical language often is presumed to comprise numerals, symbols, and letters, but a large part of the language is words, or mathematics vocabulary. Mathematics vocabulary knowledge is crucial for the overall development of mathematical proficiency (Riccomini et al., 2015). We define *mathematics vocabulary* as vocabulary terms essential for the understanding of mathematics concepts and procedures. Students need mathematics vocabulary to effectively communicate and reason through verbal and written language. However, mathematics vocabulary often is complex and difficult for students because vocabulary terms may have different meanings across disciplines (e.g., *inequality, factors*) or within mathematics (e.g., *degrees* of an angle and the *degrees* of temperature), or are homophones (e.g., *some* and *sum*), just to name a few. Therefore, teachers should provide mathematics vocabulary instruction to strengthen students' mathematics vocabulary knowledge, enhance their engagement with mathematical concepts and procedures, and improve problem solving.

Research Supporting the Practice

Mathematics vocabulary can be understood in terms of four categories: *General, technical, subtechnical,* and *symbolic* (Monroe & Panchyshyn, 1995). General vocabulary is everyday language terms used to describe, explain, and discuss. Mathematics-specific vocabulary (i.e., technical, subtechnical, and symbolic), on the other hand, entails various degrees of context-specific definitions. Technical vocabulary terms only have one meaning (e.g., *polynomial*); subtechnical vocabulary terms have more than one meaning as mentioned previously (e.g., *factor, degrees, sum*); and symbolic vocabulary include numerals, symbols, and abbreviations (e.g., x^2). Students need to know the meanings in the various mathematical contexts to grasp concepts and problem-solve. To help students, researchers have recommended explicitly teaching mathematics vocabulary to promote conceptual and procedural understanding in mathematics, especially as mathematics language becomes increasingly more sophisticated and challenging at higher grades (Dunston & Tyminski, 2013; Powell et al., 2019).

Mathematics vocabulary knowledge also can alleviate cognitive load and reduce misunderstandings or misconceptions. Recent academic reviews revealed that mathematics vocabulary knowledge plays an essential role in facilitating cognitive processes, such as reasoning and word problem-solving (Lin et al., 2021; Peng et al., 2020). For example, Lin et al. (2021) learned that mathematics vocabulary knowledge is positively associated with mathematics performance and higher-order mathematical tasks such as problem-solving, fractions, and algebra. When students have a strong mathematics vocabulary, they are better equipped to allocate cognitive resources to interpret and solve complex problems, rather than struggling to understand terminology. For example, if students understand the meaning of the terms *coefficients* and *factoring*, they will know the relation between the two and be able to simplify or operate algebraic expressions, equations, and inequalities.

Moreover, vocabulary instruction is favorable for both English learners and students experiencing mathematics difficulty (Fuchs, Seethaler, et al., 2021; Stevens et al. 2024). A study showed that students with mathematics difficulty who received vocabulary instruction outperformed students who did not receive vocabulary instruction in word-problem solving (Stevens et al. 2023). Consistent with Peng et al.'s (2020) framework that asserts the role that language has over the communication and retrieval of mathematics knowledge, students who can readily access their mathematics vocabulary repertoire are better positioned to understand word problems and solve such problems successfully.

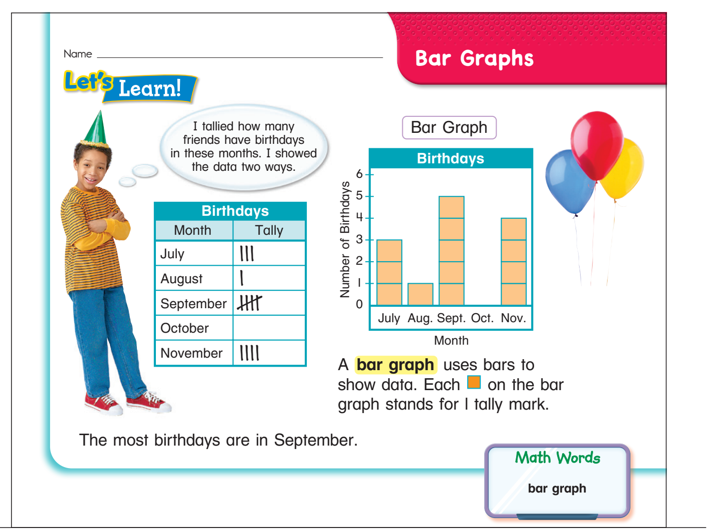
Mathematics Vocabulary Instruction in Action

There are two key components of mathematics vocabulary instruction. First, students should be exposed to and understand the formal language of mathematics (Hughes et al., 2016). In the early elementary grades, there are approximately 100–150 formal vocabulary terms that students might encounter in the mathematics classroom. In the late elementary grades, students may be responsible for knowing 300–400 terms. By middle school, students may interact with over 500 terms (Powell, Bos, et al., 2021). Teachers must emphasize the formal language of mathematics by using (and giving students an opportunity to use) formal terms. Therefore, instead of saying *answer*, teachers and students should talk about the *sum, difference, product,* or *quotient*. Instead of saying *top number*, students should say *numerator*. Instead of saying *corner*, students should use *vertex*.

Second, students should have a precise understanding of the definitions of vocabulary terms (Powell et al., 2019). Many times, students may know the formal terms (e.g., *factor, multiple*), but they may not have a precise definition for the terms. In such cases, students may use terms interchangeably and show limited understanding with the application of terms (e.g., in $3 \times 2 = 6$ describing 3 and 2 as *multiples* instead of *factors*).

Teachers can use representations (e.g., hands-on tool, virtual manipulatives, drawings) to help students understand the meaning of mathematics vocabulary terms.

Teachers can support the development of students' formal mathematics vocabulary terms and definitions through several instructional practices. Lariviere, Arsenault, et al. (2024) identified explicit instruction, use of representations, use of graphic organizers, pre-teaching, and repeated exposures as important for helping students develop a deep understanding of mathematics vocabulary terms. We already have reviewed explicit instruction in this paper. Teachers can use representations (e.g., hands-on tool, virtual manipulatives, drawings) to help students understand the meaning of mathematics vocabulary terms. Many times, word walls or similar vocabulary resources include the vocabulary term, a precise definition, and a representation to help students connect meaning to the term. Graphic organizers also could be used to help student explore terms, their definitions, examples, and non-examples (Stevens et al., 2023). Pre-teaching would involve highlighting key vocabulary terms and corresponding definitions before teaching mathematics content while repeated exposures would involve providing students many opportunities to practice speaking, reading, hearing, and writing mathematics vocabulary terms and their definitions.



***Progress in Mathematics*, Grade 2**

Fluency Practice

Students demonstrate fluency in mathematics when they can solve problems accurately and efficiently. We define *fluency* as ease and accuracy with mathematics concepts and procedures. As students develop fluency, their cognitive resources are freed up for problem solving and more complex tasks (Chandler & Sweller, 1991). Fluency activities should not be focused on rote memorization, but rather should support students in developing conceptual and procedural understandings of the skill. Fluency in mathematics begins with counting, progresses to single-digit facts, and builds to problem-solving tasks like factoring equations or solving word problems.

Research Supporting the Practice

Counting is a significant early milestone for students' fluency and is a predictor of students' later mathematics performance (Classens & Engel, 2013; Koponen et al., 2019). When teaching counting, it is important to ensure students demonstrate mastery across all five counting principles: stable order, one-to-one correspondence, cardinality, abstraction, and order-irrelevance (Frye et al., 2013). Students with counting fluency understand that the following number is exactly one more than the previous number (i.e. 12 is one more than 11) and begin to develop their mental number line. Mathematics instruction incorporating multiple representations (e.g., the number line or counters) can be a useful strategy to help students who struggle with counting fluency (Dyson et al., 2015; Fuchs, Geary, et al., 2013).

After developing proficiency with counting, students need to develop fluency with mathematics facts, including addition facts with single-digit addends, subtraction facts with single-digit subtrahends, multiplication facts with single-digit factors, and division facts with single-digit divisors. Students with mathematics difficulty often struggle with mathematics fact fluency because of difficulty retrieving mathematics facts from long-term memory or procedural errors (Geary, 2004). These students also require more attempts and time to develop proficiency with mathematics facts compared to their peers without mathematics difficulty (Stickeney et al., 2012). As such, brief and daily practice is recommended for these students. Incorporating modeling into activities aids in students' conceptual development of fact fluency (Codding et al., 2011). Early mathematics instruction with fact fluency is critical for ensuring these difficulties do not persist into high school (Calhoun et al., 2007).

Fluency with mathematics facts builds to students' success with rational numbers (i.e. fractions), word-problem solving, and algebra (Powell et al., 2023; Seethaler et al., 2011; Tolar et al., 2009). These higher-level mathematics skills rely on fluency where students apply algorithms or procedures flexibly and accurately. This fluency allows students to generalize their knowledge of mathematics to new problems or situations, which is increasingly important as students participate in high-stakes standardized assessments (National Council for Teachers of Mathematics, 2023). For students to be successful in the mathematics classroom and on high-stakes assessments, they must have strong mathematics fact fluency and demonstrate fluency when solving a variety of problems. To support students who struggle with fluency, teachers need to provide opportunities for guided and independent practice where students receive frequent feedback (Suh & Moyer-Packenham, 2007).

Fluency Practice in Action

Counting bears, cars, or other manipulatives are useful tools to support students' counting fluency. When using counters, model and explicitly teach counting and comparison to students. Teachers also can group different types of objects (e.g., bears and cars) to demonstrate that a set does not have to be all the same type. Mathematics instruction should focus on the earliest stage of students' difficulty and progress when students demonstrate proficiency.

Fact fluency practice can be supported through a variety of games and activities such as flashcard activities or the dice game where students roll the dice then add, subtract, multiply, or divide the numbers. As students begin to practice their fact fluency, students should learn strategies to support their computation, such as the use of counting up strategies or a multiplication table. Additionally, teachers should sequence mathematics fact practice so students begin with easier facts and progress to harder ones. Graphs or reward systems that incentivize students to answer more facts correctly can be an effective motivation for brief and daily fact fluency practice. The goal of fluency practice is for students to eventually develop automaticity with their mathematics facts and minimize their reliance on back-up strategies like counting.

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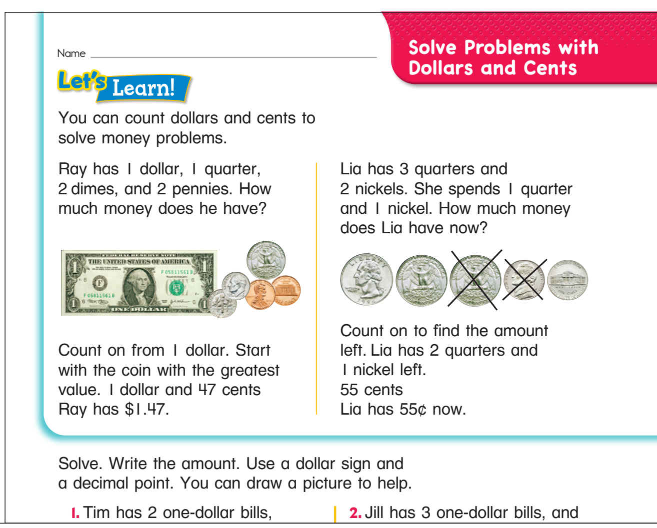
For more advanced mathematics skills, fluency practice is not intended for memorization, but rather for students to complete mathematics problems with accuracy. Depending on the skill of interest, instructional practices will vary. For skills related to computation or manipulations of equations, teaching additional strategies may be more effective for helping students develop procedural fluency. For example, if a student is struggling with the standard addition algorithm, teaching the partial sums strategy may help them develop addition fluency. Similarly, students could use the box method or algebra tiles to multiply binomials. For word-problem solving, teaching an attack strategy and schema instruction provide students with a generalized process for solving all problems (Powell & Fuchs, 2018). These practices, which we detail in the next section of this chapter, help students develop word-problem fluency by providing a step-by-step process that breaks apart a complex process.

Problem Solving

National education organizations significantly value the teaching of problem solving across grades K–12 (e.g., National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), and high-stakes standardized tests strongly emphasize mathematics word problems (e.g., National Assessment Governing Board, 2017) with over 95% of high-stakes items involving the solving of word problems (Powell, Namkung, et al., 2022). We define *word problems* as a scenario presented with words and numbers that requires students to interpret the prompt or question, provide a response, and connect the abstract to real-world


Name

Solve Problems with Dollars and Cents

Let's Learn!

You can count dollars and cents to solve money problems.

Ray has 1 dollar, 1 quarter, 2 dimes, and 2 pennies. How much money does he have?

Count on from 1 dollar. Start with the coin with the greatest value. 1 dollar and 47 cents
Ray has $1.47.

Lia has 3 quarters and 2 nickels. She spends 1 quarter and 1 nickel. How much money does Lia have now?

Count on to find the amount left. Lia has 2 quarters and 1 nickel left.
55 cents
Lia has 55¢ now.

Solve. Write the amount. Use a dollar sign and a decimal point. You can draw a picture to help.

1. Tim has 2 one-dollar bills,

2. Jill has 3 one-dollar bills, and

***Progress in Mathematics*, Grade 2**

Teachers should consider embedding problem solving into their daily mathematics lessons to ensure students are adequately prepared to set up and solve a variety of word problems.

experiences (Depaepe et al., 2010). To demonstrate proficiency with problem solving, students need to develop a set of skills and strategies for solving a range of word problems. Because reading, language, and the concepts and procedures of mathematics serve as prerequisite skills for understanding word problems, many students, especially those with mathematics difficulty, experience challenges when solving word problems (Jitendra & Star, 2011; Krawec et al., 2012; Xin et al., 2005). Teachers should consider embedding problem solving into their daily mathematics lessons to ensure students are adequately prepared to set up and solve a variety of word problems.

Research Supporting the Practice

Word problems typically fall into three distinct categories: *directive word problems, routine word problems,* and *non-routine word problems*. Directive word problems provide directions to complete a task or find missing information. Many directive word problems relate to algebra, geometry, and measurement (e.g., *Which figure shows a line of symmetry? Which expression is equivalent to 8(2a + 1b)*?). Routine word problems present numbers within a problem, or within a chart or graph, and include a word-problem prompt or question that encourages students to manipulate the numbers to find the answer. Routine word problems may involve one or more steps. An example routine word problem is: *A cookie recipe requires 1.5 cups of sugar for every batch of cookies. If Tara baked 4 batches of cookies, how many cups of sugar did she use?* Non-routine word problems usually include multiple answers or multiple ways to solve a problem (e.g., *Three cartons have capacities of 3.5 pints, 4.5 pints, and 6 pints. How can you use these three cartons to measure exactly 11 pints of milk?*). Students need to know how to set up and solve all three types of word problems: directive, routine, and non-routine.

Competency in early word-problem solving has major implications for later mathematical learning and postsecondary opportunities (Baum et al., 2010; Ritchie & Bates, 2013), which validates the need for teachers to place a strong emphasis on teaching problem solving within elementary, middle, and high school. Fortunately, a strong research base exists to help teachers understand how to provide effective word-problem instruction to students (Powell, Berry, et al., 2021; Jitendra et al., 2002; Jitendra & Star, 2012; Van de Walle et al., 2013; Xin & Zhang, 2009).

Many teachers across the United States rely on defining problems by operation (e.g., *This is a subtraction problem*) and tying key words to operations (e.g., *altogether means add; each means divide*). Teaching students to approach problems as *addition problems* or *division problems* hinders students' reasoning abilities. Moreover, tying key words to operations works less than 10% of the time once students are exposed to multi-step problems (Powell, Namkung, et al., 2022). Unfortunately, these approaches do not promote students' conceptual understanding, and neither strategy has research evidence to support its use (Powell & Fuch, 2018). In contrast, there are two research-validated practices for teaching word-problem solving to students spanning all grade levels: the use of an attack strategy and schema instruction (Fuchs et al., 2014; Montague, 2008; Powell, Berry, et. al, 2021; Xin & Zhang, 2009). First, teachers should use an attack strategy to teach students to set up and solve word problems. An attack strategy is a generalized process for setting up and solving a word problem, often including a mnemonic device or acronym for easy recall. Grounded in metacognitive strategies, an attack strategy often includes an element of self-regulation in which students ask themselves questions and self-monitor their steps as they work through a problem. Frequently-used attack strategies include UPS Check (Understand, Plan, Solve, and Check; adapted from Pólya, 1945), FOPS (Find the problem, Organize information using a diagram, Plan to solve the problem, and Solve the problem; Jitendra & Star, 2012), and STAR (Search the word problem, Translate the words into an equation or picture, Answer the problem, and Review the solution; Gagnon & Maccini, 2001). With any attack strategy, students need explicit instruction with modeling and practice to apply the attack strategy to directive, routine, and non-routine word problems. Teachers may choose any attack strategy provided (1) the first step involves reading the problem and (2) the same attack strategy is used to solve all word problems. As students become more advanced problem solvers, they will rely less on the attack strategy as the process will become more engrained.

Second, teachers should teach word-problem schemas, also termed word-problem structures or word-problem types, in tandem with the attack strategy to support students' problem-solving skills. When using schemas, students first identify a word problem as belonging to a problem type and then use a specific solution strategy associated with that schema to solve the problem. Identifying word problems as belonging to a specific schema has proven to be an effective word-problem strategy for students (e.g., Powell, Berry, et al., 2020; Powell, Berry, et. al, 2021; Flores et al., 2016; Fuchs et al., 2014; Jitendra et al., 2013). There are six schemas that can be used to solve word problems from kindergarten through eighth grade: total, difference, change, equal groups, comparison, and ratios and proportions.

The three additive schemas, total, difference, and change, involve addition or subtraction procedures. In total problems, also termed combine or part-part-whole problems, students put parts together for a total or sum. Students can use the total equation $P + P = T$ *(part plus part equals total)* to set up and solve any total problem. In difference problems, also called compare problems, two amounts are compared for a difference. Students can use the difference equation $G - L = D$ *(the greater amount minus the lesser amount equals the difference)* to set up and solve any difference problem. In change problems, sometimes called join or separate problems, a starting amount increases or decreases to a new amount. Students can use the change equation

$ST + C = E$ *(the starting amount plus the change amount equals the end amount)* to solve any change increase problem and the change equation $ST - C = E$ *(the starting amount minus the change amount equals the end amount)* to solve any change decrease problem. See the figure below for example word problems and accompanying equations across the three additive schemas.

Schema	Word Problem Example 1	Equation Example 1	Word Problem Example 2	Equation Example 2
Total	*The store has 112 fireman hats and 83 police hats. How many hats does the store have?*	$P + P = T$ $112 + 83 = ?$	*The store has 195 fireman hats and police hats. If there are 112 fireman hats, how many police hats are there?*	$P + P = T$ $112 + ? = 195$
Difference	*The store has 112 fireman hats and 83 police hats. How many more fireman hats does the store have (How many fewer police hats)?*	$G - L = D$ $112 - 83 = ?$	*The store has 29 more fireman hats than police hats. If there are 83 police hats, how many fireman hats are there?*	$G - L = D$ $? - 83 = 29$
Change	*The store had 83 hats. Then they bought some more hats. Now, there are 112 hats. How many hats did the store buy?*	$ST + C = E$ $83 + ? = 112$	*The store had some hats. Then they sold 112 hats, and there are 83 left. How many hats did the store have to start?*	$ST - C = E$ $? - 112 = 83$

The three multiplicative schemas, equal groups, comparison, and ratios and proportions, involve multiplication and division procedures. In equal groups problems, students make groups with an equal number in each group. Students can use the equal groups equation $GR \times N = P$ *(groups times the number in* *each group equals the product)* to set up and solve any equal groups problem. In comparison problems, a set is multiplied a number of times for a product. Students can use the comparison equation $S \times T = P$ *(the set times the multiplier or number of times equals the product)* to set up and solve any comparison problem. In ratios and proportions problems, which commonly are introduced in Grade 6, students identify relationships among quantities. The ratios and proportions schema includes all word problems related to ratios, proportions, percentages, and unit rate. Students can use the ratios and proportions equation

$\frac{Q1}{Q2} = \frac{Q3}{Q4}$ *(quantity one is to quantity two as quantity three is to quantity four)* to set up and solve any ratios and proportions problem. See the figure below for example word problems and accompanying equations across the three multiplicative schemas.

Schema	*Word Problem Example 1*	*Equation Example 1*	*Word Problem Example 2*	*Equation Example 2*
Equal Groups	*The store has 5 hats. There are 12 polka dots on each hat. How many polka dots are on all of the hats?*	$GR \times N = P$ $5 \times 12 = ?$	*The store has 195 hats. 5 customers bought all of the hats. If each customer bought the same number of hats, how many hats did each customer buy?*	$GR \times N = P$ $5 \times ? = 195$
Comparison	*The store has 83 police hats. There are 5 times as many fireman hats. How many fireman hats are there?*	$S \times T = P$ $83 \times 5 = ?$	*There are 5 times as many fireman hats as police hats. If there are 415 fireman hats, how many police hats are there?*	$S \times T = P$ $? \times 5 = 415$
Ratios and Proportions	*The store sold 5 hats in 15 minutes. At this rate, how many hats will the store sell in 25 minutes?*	$\frac{Q1}{Q2} = \frac{Q3}{Q4}$ $\frac{5}{15} = \frac{?}{25}$	*The store sold police hats and fireman hats. The ratio of police hats sold to fireman hats sold was 2:5. If the store sold 83 police hats, how many fireman hats did the store sell?*	$\frac{Q1}{Q2} = \frac{Q3}{Q4}$ $\frac{2}{5} = \frac{83}{?}$

As teachers introduce the six schemas, they also need to model and provide students an opportunity to practice solving multi-step word problems that include two or more schemas. For example, students might see a multi-step total and difference problem (e.g., *Ms. Powell sold 30 necklaces during Week 1 and 55 necklaces during Week 4. She sold 40 necklaces during Week 5. How many more necklaces did Ms. Powell sell in Weeks 1 and 4 than in Week 5?*). In this example, students can apply the total schema with a missing total to determine the number of necklaces sold during weeks 1 and 4 ($30 + 55 = ?$), and then apply the difference schema with a missing difference to determine how many more necklaces were sold during weeks 1 and 4 than in week 5 ($85 - 40 = ?$). Students also may be exposed to other multi-step combinations such as multi-step equal groups and equal groups problems where they need to multiply twice, multiply once and divide once, or divide twice to generate the correct solution. For example, *Ms. Hardy bought 10 bags of chocolate candies. Each bag contained 5 chocolate candies. She put an equal number of chocolate candies into 2 jars. How many chocolate candies are in each jar?* In this problem, students can apply the equal groups schema with a missing product to determine the total number of chocolate candies ($10 \times 5 = ?$), and then the equal groups schema with a missing number in each group ($2 \times ? = 50$) to determine the number of chocolate candies in each jar. Other common multi-step schema combinations include total and equal groups as well as equal groups, equal groups, and total or difference. Teachers need to provide explicit instruction on setting up and solving a variety of single- and multi-step word problems.

Problem Solving in Action

Teachers can support students' understanding of an attack strategy and word-problem schemas through several instructional practices. Initially, teachers need to utilize the tenets of explicit instruction to teach an attack strategy and the schemas. Teachers need to provide clear modeling with explanations, include numerous practice opportunities with planned examples and non-examples, and offer frequent affirmative and constructive feedback. Teachers also need to introduce one schema at a time, beginning with the total schema. After students understand the total schema, teachers can introduce the difference schema, and then the change schema. Once students have developed an understanding of the three additive schemas, teachers can transition to the multiplicative schemas. As teachers introduce each schema, they need to teach students to use an equation to organize the word-problem information and solve for the unknown, as displayed in the equation examples in the two figures above.

To ensure students are successful with problem solving, teachers also need to provide necessary background knowledge and teach important mathematics vocabulary terms. Teachers cannot assume that students, especially English learners, understand frequently-used mathematics word-problem terms (e.g., *percentage, rate, area, length, perimeter, cost*) or have fluency with concepts of money, time, addition and subtraction regrouping, and multiplication and division facts. Teachers need to explicitly teach these vocabulary terms and embed daily fluency practice into each problem-solving lesson to promote students' optimal success.

The schema sorting activity provides another opportunity to help students understand the schemas. In the schema sorting activity, students do not solve the word problem. Instead, they determine the schema or problem type and sort the word problem into the correct schema category (e.g., total, difference, change, equal groups, comparison, or ratios and proportions) on a sorting mat. The sorting activity provides practice for students to distinguish among the six schemas.

Another way to enhance students' understanding of an attack strategy and schemas involves having students create their own word problems. When students generate their own word problems, they develop a deeper understanding of the schemas. Students can solve the word problems created by their peers, which maximizes student engagement and provides an opportunity for embedded mathematics-writing practice. Lastly, teachers can post a word problem of the day on the board to expose students to directive, routine, and non-routine word problems they are likely to see on high-stakes assessments.

Conclusion

When designing and delivering mathematics instruction, teachers should rely upon practices that have research to support their use (i.e., research-validated practices). In this guide, we reviewed four research-validated practices. Often, teachers might use all four of these practices within a single class period. With *explicit instruction*, teachers model mathematics and engage students in guided practice and independent practice. During modeling and practice, teachers support student engagement by asking high- and low-level questions, eliciting frequent responses, providing affirmative and corrective feedback, and maintaining a brisk pace. With *mathematics vocabulary instruction*, teachers model and practice formal mathematics vocabulary terms and ensure students understand the precise meaning of each term. With *fluency practice*, teachers provide multiple practice opportunities for students to develop fluency—ease and accuracy—with solving mathematics problems. With *problem solving*, teachers pair an attack strategy with schemas to help students set up and solve a variety of word problems.

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